Secant-Method Adjustment for Structural Models

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On-orbit testing will be required for final tuning and validation of any mathematical model of large space structures. Identification methods using limited response data to produce optimally adjusted property matrices seem ideal for this purpose, but difficulties exist in the application of previously published methods to large space truss structures. This article presents new stiffness matrix adjustment methods that generalize optimal-update secant methods found in quasi-Newton approaches for nonlinear optimization. Many aspects of previously published methods of stiffness matrix adjustment may be better understood within this new framework of secant methods. One of the new methods preserves realistic structural connectivity with minimal storage requirements and computational effort. A method for systematic compensation for errors in measured data is introduced that also preserves structural connectivity. Two demonstrations are presented to compare the new methods' results to those of previously published techniques.

Nomenclature

$\|B\|_F$ = Frobenius matrix norm of an $m \times n$ matrix $[B]$, $= \left[\sum_{i=1}^{m} \sum_{j=1}^{n} (B_{ij})^2 \right]^{1/2}$
B_{ij}^c = ijth element of the matrix $[B_c]$
$(B)_j$ = jth element of $\{B\}$
$[D]$ = $n \times n$ diagonal weighting matrix
d_i = ith diagonal element of $[D]$, $= \sqrt{K_{ii}^c}$
$[E]$ = $n \times n$ error matrix, $= [K] - [K_c]$
$[F]$ = $n^2 \times np$ block diagonal matrix
$[I]$ = identity matrix
$[K]$ = $n \times n$ stiffness matrix
$[K_c]$ = $n \times n$ original model stiffness matrix
$[K_+]$ = $n \times n$ adjusted stiffness matrix
$[M]$ = $n \times n$ mass matrix
n = number of degrees of freedom
$[P]_i$ = $n \times n$ diagonal matrix of ones and zeros, which masks a vector with the sparsity pattern of the ith row of $[K_c]$
p = number of measured modes
$[S]$ = $n \times p$ matrix of p mode shape vectors
$S_{jk}^{(i)}$ = jkth element of $[S]$ masked with the zero-nonzero pattern of the ith row of $[K_c]$
sparse (B) = zero-nonzero pattern of a matrix $[B]$
$[T]$ = $n \times n$ storage matrix
$[Y]$ = $[M][S][\Omega^2] - [K_c][S]$
$[Z]$ = $np \times np$ diagonal matrix
$\{\Gamma\}$ = $np \times 1$ partitioned vector of Lagrange multipliers
$\{\Gamma_i\}$ = ith $p \times 1$ subvector of $\{\Gamma\}$
$\{\Delta\}$ = $np \times 1$ partitioned vector
$\{\Delta_i\}$ = ith $p \times 1$ subvector of $\{\Delta\}$
λ_{ik} = ikth Lagrange multiplier
μ = a uniformly distributed random variable between ± 0.10
$[\Pi]$ = $n^2 \times n^2$ permutation matrix that converts a columnwise listing of an $n \times n$ matrix to a columnwise listing of its transpose, $[\Pi][z_{11}z_{12}\ldots z_{1n}z_{21}\ldots z_{2n}\ldots z_{nn}]^t = [z_{11}z_{21}\ldots z_{n1}z_{12}\ldots z_{nz}\ldots z_{nn}]^t$
$[\Sigma]$ = $p \times p$ diagonal weighting matrix
σ_k = kth diagonal element of $[\Sigma]$
ϕ_i = ith column of $[T] + [T]^t$
$[\Omega^2]$ = $p \times p$ diagonal matrix of squared circular frequencies

Introduction

OPTIMAL-UPDATE stiffness matrix adjustment methods have several qualities that promote their application for on-orbit identification of large space structures. Computations are performed off line, and the use of an original model stiffness matrix that is close to correct provides a great deal of a priori information that reduces the need for extensive on-orbit data. However, difficulties have been encountered with the use of these methods that would prohibit their application to large space structures such as the space station. This article summarizes the development and testing of a number of new methods for optimal stiffness matrix adjustment. Performance comparable to the best of the previously published optimal-update identification methods has been observed and is reported here. As an added benefit, the point of view we adopt here provides insight into the workings of some previously known matrix adjustment strategies.

Structural model identification has received considerable attention in recent years. In particular, stiffness matrix adjustment methods, which use measured response data to produce an optimal update of the stiffness matrix, have been the focus of much research.[1-4] These optimal-update identification methods have been applied to problems of model correlation[5,6] and have also shown potential for application to problems of damage location in large space truss structures.[7,8]

Baruch and Bar Itzhack,[1] in their work on orthogonalization of measured modes, developed an optimal update for the stiffness matrix for a structure. Measured frequencies and mode shapes are used to modify an initial stiffness matrix, finding the closest matrix that is consistent with the observed modal data. Berman and Nagy[2] adopted the same approach in their method to improve analytical models using test data and addressed the task of finding mass matrix updates as well. An extension of Berman's method has been applied to problems of test/analysis correlation.[5]

With Baruch and Bar Itzhack's method, the adjusted stiffness matrix often contains nonzero elements where zeros ex-

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isted in the original matrix. If the original matrix corresponds to a reduced model with few zero elements, this might not be a difficulty. However, for the similar adjustment of an original finite-element model stiffness matrix having considerable sparsity, the final storage requirement of the dense result may prohibit further processing. Worse, the nonzero elements created in the adjusted stiffness matrix correspond to physical load paths that do not exist in the actual structure, thus reducing confidence in the adjusted model.

Kabe[3,4] addressed these concerns and presented a stiffness matrix adjustment procedure that preserves the physical connectivity of the original model in the updated stiffness matrix. However, a large, indefinite auxiliary linear system of equations must be solved as a consequence. Application of Kabe's method is limited by the storage and computational effort required,[9] but, nonetheless, the method has been successful for model correlation[6] and damage location[7,8] in problems of modest extent.

Kammer[15] presented a stiffness matrix identification method with the same objectives as in Kabe's work. Although a different formulation was used to develop the update, the results are the same as Kabe's in many circumstances. As Kammer notes, for large problems it appears that this method has practical shortcomings similar to Kabe's method.

Many of the issues that are faced in optimal stiffness matrix adjustment for structural model identification have immediate analogs in optimal secant updates used in quasi-Newton methods for nonlinear optimization. This observation has motivated and aided much of the work we present here and has provided a unifying point of view for much previous work. For example, the Baruch/Bar Itzhack update of the stiffness matrix may be viewed directly as a multiple-secant generalization of the Davidon-Fletcher-Powell (DFP) update. Details of this development are presented by the authors in Ref. 11 and are summarized later. Independently, Schnabel[12] has developed some similar multiple-secant methods motivated by the need to accelerate quasi-Newton methods implemented on parallel computer architectures.

Baruch and Bar Itzhack directly solved a constrained optimization problem to find the symmetric stiffness matrix that is closest to an original stiffness matrix and that satisfies the dynamic constraints imposed with the measured data. The problem is formulated as

$$\min_{[K]} \|[M]^{-1/2}([K]-[K_c])\,[M]^{-1/2}\|_F \tag{1}$$

subject to $[K][S] = [M][S][\Omega^2]$ and $[K] = [K]^t$

The constrained optimization problems lead to a closed-form solution requiring only matrix operations. The updated stiffness matrix is found as

$$\begin{aligned}[K_+] = [K_c] &- [K_c][S][S]^t[M] - [M][S][S]^t[K_c] \\ &+ [M][S][S]^t[K_c][S][S]^t[M] \\ &+ [M][S][\Omega^2][S]^t[M]\end{aligned} \tag{2}$$

In the study of quasi-Newton methods for nonlinear optimization,[10] secant update methods produce an adjusted Hessian matrix, which satisfies a single constraint, called the secant condition. The first step in comparing secant update methods to stiffness matrix adjustment methods is to generalize the secant method to formulation with multiple constraints. In Ref. 11, the DFP secant update is modified to include multiple constraints and provides a solution to

$$\min_{[A]} \|[T]^{-t}([A]-[A_c])\,[T]^{-1}\|_F \tag{3}$$

subject to $[A][S] = [X]$ and $[A] = [A]^t$

where $[A_c]$ is the original matrix, $[T]$ is an $n \times n$ weighting matrix, and $[S]$, $[X]$ are $n \times p$ matrices in the p constraints with $[S]^t\,[X]$ symmetric and nonsingular. The Baruch/Bar Itzhack method is an immediate consequence of setting $[X] = [M][S][\Omega^2]$ and taking $[T] = [M]^{1/2}$. Therefore, the Baruch/Bar Itzhack update of the stiffness matrix is a multiple-secant generalization of the DFP update.

Two other secant methods were generalized with multiple constraints to produce stiffness matrix update methods. One of these new methods preserves structural connectivity with minimal storage and computational effort. A further extension permits compensation for errors in the measured data. After presenting the theoretical development of these methods, this article includes two comparisons of the performance of the new methods to that of previously published techniques.

Theoretical Development

The similarity of the Baruch/Bar Itzhack stiffness matrix update technique and a multiple-secant generalization of the DFP method has led to insight into currently used optimal-update identification methods and to the development of new structural identification techniques. Improved understanding of optimal-update identification follows from recognizing the connection to secant methods because the latter have been investigated for nearly twice as long as the former. For example, published studies of the DFP method prove that positive definiteness is guaranteed in the adjusted matrix (updated with a single constraint) whenever the original matrix is positive definite. Analogously, Ref. 11 provides a proof that the adjusted stiffness matrix generated by the Baruch/Bar Itzhack method is positive definite so long as the original matrix is positive definite.

A more important benefit of this framework, however, is that new methods of stiffness matrix adjustment can be developed as extensions of secant update methods. The first new method is a stiffness matrix modification technique based on finding the closest flexibility matrix that is consistent with the observed modal data. It is derived by a multiple-secant generalization of the update associated with Broyden, Fletcher, Goldfarb, and Shanno (BFGS).[11] The solution to

$$\min_{[K]} \|[M]^{1/2}([K]^{-1}-[K_c]^{-1})\,[M]^{1/2}\|_F \tag{4}$$

subject to $[K][S] = [M][S][\Omega^2]$ and $[K] = [K]^t$

is given by

$$\begin{aligned}[K_+] = [K_c] &+ [M][S][\Omega^2][S]^t[M] \\ &- [K_c][S]([S]^t[K_c][S])^{-1}[S]^t[K_c]\end{aligned} \tag{5}$$

Again, a positive definite original stiffness matrix $[K_c]$ produces a positive definite update.

The multiple-secant DFP and BFGS methods (denoted MS-DFP and MS-BFGS) both produce a stiffness matrix that loses the zero-nonzero pattern of the original matrix, thus losing representation of the structure load paths. This characteristic of Baruch and Bar Itzhack's method promoted Kabe[3,4] to include structural connectivity information in an optimal stiffness matrix adjustment method. An updated stiffness matrix that is closest to the original matrix, yet satisfies constraints imposed by the modal data, symmetry, and the zero-nonzero pattern of the original matrix, is found with the measure of closeness as a sum of the change percentage for each nonzero element of the stiffness matrix

$$\sum_{i=1}^{n} \sum_{j \epsilon \mathrm{adj}(i)} \frac{(K_{ij}-K_{ij}^c)^2}{K_{ij}^{c\,2}} \tag{6}$$

The notation $j \epsilon \text{adj}(i)$ indicates that this summation is for all nonzero elements of the original stiffness matrix, elements with j in the adjacency set of i.

Preservation of the zero-nonzero pattern of the original stiffness matrix enables successful identification of large structural models with relatively few measured modes. Unfortunately, an auxiliary problem must be solved for this method. Considerable storage and computational effort are required to solve the auxiliary system of equations, which is indefinite and has a dimension equal to the number of degrees of freedom times the number of modes used for the identification.

Sparsity is also preserved in the secant update method of Marwil[13] and Toint.[14] Generalization to a multiple-secant algorithm produced an optimal-update identification method that overcomes the difficulties of Kabe's method while maintaining the benefits. In the multiple-secant Marwil-Toint (MSMT) method,

$$\sum_{i,j=1}^{n} \frac{(K_{ij} - K_{ij}^c)^2}{K_{ii}^c K_{jj}^c} \tag{7}$$

is minimized,

$$\text{subject to } [K][S] = [M][S][\Omega^2], \qquad [K] = [K]^t \tag{8}$$

and sparse $([K])$ = sparse $([K_c])$

This objective function, restated for comparison with the MS-DFP method, is

$$\|[D]^{-1}([K] - [K_c])[D]^{-1}\|_F^2 \tag{9}$$

with

$$[D] = \text{diag}\,(d_i) = \text{diag}\,(\sqrt{K_{ii}^c})$$

Lagrange multipliers incorporate the constraints into an extended cost function. Minimization of the Lagrangian function produces a system of linear equations to solve for the Lagrange multipliers and an update equation for the stiffness matrix. The auxiliary problem has the same $np \times np$ dimension as Kabe's problem, but is symmetric and positive semidefinite, rather than indefinite, permitting solution without the large storage requirement and with less computational effort. Details of the MSMT method derivation are presented in the Appendix.

With the solution of the auxiliary problem, elements of the adjusted stiffness matrix are formed from the original stiffness matrix as

$$\begin{aligned} K_{ij}^+ &= K_{ij}^c + d_i d_j\, [([P]_i[D][S]\{\Gamma_i\})_j \\ &\quad + ([P]_j[D][S]\{\Gamma_j\})_i] \qquad \text{for } i,j = 1,2\ldots, n \end{aligned} \tag{10}$$

The diagonal matrix $[P]_i$ contains only ones and zeros to mask the mode shape vectors in $[S]$ with the sparsity pattern of the ith row of $[K_c]$. The sparsity pattern of $[K_c]$ does not consider zeros produced by structural symmetry (force cancellation), but rather represents physical connections in the structure between degrees of freedom of the model.

The partitioned vector $\{\Gamma\}$ is the solution of the auxiliary system of equations, constructed from the original model and measured modal data as

$$[F]^t([I] + [\Pi])[F] \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{bmatrix} \tag{11}$$

where $[F]$ is a block diagonal matrix of weighted masked modal vectors using the same projection matrix $[P]_i$,

$$[F] = \text{diag} \begin{bmatrix} [P]_1[D][S] & & & \\ & [P]_2[D][S] & & \\ & & \ddots & \\ & & & [P]_n[D][S] \end{bmatrix} \tag{12}$$

which is combined with the identity matrix $[I]$ and a reordering matrix $[\Pi]$ to form the auxiliary problem coefficient matrix. The right-side vector $\{\Delta\}$ is another partitioned vector formed with the given data

$$\{\Delta_i\} = d_i^{-1} \begin{bmatrix} Y_{i1} \\ Y_{i2} \\ \vdots \\ Y_{ip} \end{bmatrix} \tag{13}$$

weighting each row of $[Y] = [M][S][\Omega^2] - [K_c][S]$.

Implementation of the method can be accomplished by assembling the full symmetric positive semidefinite system and solving for $\{\Gamma\}$. However, the dimension of the system $(np \times np)$ can exceed computer storage capabilities if a large structure is considered. Iterative methods for solving Eq. (11) take advantage of the repetitive substructure patterns and never assemble the coefficient matrix explicitly. A conjugate gradient method with diagonal preconditioning was used for solution of the auxiliary problem and required no more storage than that for the original stiffness matrix. The Appendix includes the details for the reduced-storage solution. Solution of the large indefinite auxiliary problem of Kabe's method involves extensive computational effort, as well as storage. Iterative solution techniques used with the MSMT update method vastly reduce the complexity of the computations required.

Performance of the MSMT method and of Kabe's method is observed to be comparable for several demonstration problems including Kabe's test problem. However, the positive semidefinite character of the auxiliary problem for the MSMT method could lead to convergence problems if the system of equations [Eq. (11)] is not consistent. Small errors in the right-side vector $\{\Delta\}$ resulting from errors in the measured modal data could produce such inconsistency.

These difficulties are circumvented with a modified MSMT approach, which is designed to compensate for measurement errors in the observed modes. The modified stiffness matrix is found to be as close as possible to the original matrix while remaining as consistent as possible with the observed modal information. The dynamic constraint is now formulated as part of the cost function as follows:

$$\begin{aligned} \min_{[K]} &\|[D]^{-1}([K] - [K_c])\,[D]^{-1}\|_F^2 \\ &+ \|([K][S] - [M][S][\Omega^2])[\Sigma]^{-1}\|_F^2 \end{aligned} \tag{14}$$

subject to $[K] = [K]^t$ and sparse $([K])$ = sparse $([K_c])$

Therefore, the second term imposes a penalty for deviating from the measured modes and frequencies, but permits deviation. Weights, σ_i in the diagonal matrix $[\Sigma]$, can be selected to assign confidence levels to the measured modes and frequencies.

The derivation of this MSMT method with error compensation (MSMT-EC) is presented in Ref. 11 along with an error compensating MS-DFP method. The resulting update equation for the MSMT-EC method is exactly Eq. (10) again.

Here, the $\{\Gamma_i\}$ are solutions to

$$\{[Z] + [F]^t([I] + [\Pi])[F]\} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{bmatrix} \tag{15}$$

where $[Z] = \text{diag}(Z_{ik})$ defined by

$$Z_{ik} = 2\left(\frac{\sigma_k}{d_i}\right)^2 \tag{16}$$

and $[F]$ and $\{\Delta_i\}$ are as previously defined in Eq. (12) and (13), respectively.

The advantageous structure of the MSMT development is seen again in the coefficient matrix of Eq. (15). Strict positive definiteness is now a result of the positive definiteness of the $[Z]$ matrix in the auxiliary problem, enabling use of classical conjugate gradient methods without the preceding concerns.

Demonstration of Algorithms

Two problems are used to demonstrate and compare the newly developed optimal-update identification methods to the previously published methods of Baruch and Bar Itzhack and of Kabe. The first problem is Kabe's example, an eight-degree-of-freedom spring-mass system. The second demonstration compares the error compensation version of the MSMT method to sparsity preserving methods without error compen-

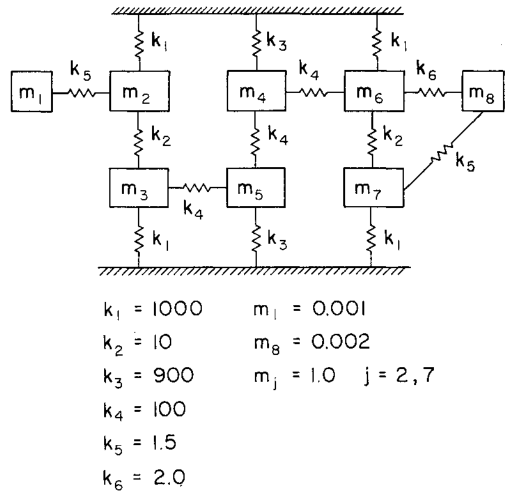


Fig. 1 Kabe's example problem.

Table 1 Stiffness element results from four identification methods

Element number	Original model	Adjusted models: MS-DFP	MS-BFGS	Kabe	MSMT	Exact model
1,1	2.00	2.00	2.00	1.50	1.50	1.50
1,2	−2.00	−3.00	−3.00	−1.50	−1.50	−1.50
1,3		−0.11	−0.10			
1,4		−0.05	−0.05			
1,5		0.08	0.06			
1,6		0.09	0.11			
1,7		−0.02	−0.02			
1,8		0.13E-03	0.16E-03			
2,2	1512.00	1024.25	1023.97	1011.50	1011.50	1011.50
2,3	−10.00	−68.51	−64.42	−10.00	−10.00	−10.00
2,4		−9.05	−11.70			
2,5		20.90	18.71			
2,6		38.48	42.57			
2,7		−9.09	−9.39			
2,8		0.06	0.07			
3,3	1710.00	1560.84	1493.53	1110.00	1110.00	1110.00
3,4		−49.83	−10.71			
3,5	−200.00	−244.10	−204.32	−100.00	−100.00	−100.00
3,6		−123.40	−189.68			
3,7		50.77	55.93			
3,8		−0.11	−0.25			
4,4	850.00	1087.90	1062.05	1100.00	1100.00	1100.00
4,5	−200.00	−25.09	4.63	−100.00	−100.00	−100.00
4,6	−200.00	−242.15	−202.95	−100.00	−100.00	−100.00
4,7		27.00	24.22			
4,8		−0.09	0.03			
5,5	850.00	1089.50	1063.69	1100.00	1100.00	1100.00
5,6		−49.50	−10.91			
5,7		−12.38	−15.61			
5,8		−0.07	0.02			
6,6	1714.00	1564.99	1499.58	1112.00	1112.00	1112.00
6,7	−10.00	−82.62	−77.60	−10.00	−10.00	−10.00
6,8	−4.00	−4.65	−4.80	−2.00	−2.00	−2.00
7,7	1512.00	1027.82	1027.41	1011.50	1011.50	1011.50
7,8	−2.00	−4.13	−4.12	−1.50	−1.50	−1.50
8,8	6.00	5.99	5.99	3.50	3.50	3.50

Three modes were used by each method for the identification. Zero element values are left blank to emphasize zero-nonzero structure.

Table 2 Comparison of eigenvalue results from four identification methods

Mode number	Original model	Adjusted models MS-DFP	MS-BFGS	Kabe	MSMT	Exact model
1	613.52	940.35	940.35	940.35	940.35	940.35[a]
2	994.11	1005.57	1003.50	1005.57	1005.57	1005.57[a]
3	1503.58	1008.25	1005.57	1008.90	1008.90	1008.90[a]
4	1510.05	1008.83	1008.83	1047.29	1047.29	1047.29
5	1745.83	1611.00	1431.29	1169.19	1169.19	1169.19
6	1765.27	1760.73	1759.96	1264.51	1264.51	1264.50
7	2008.07	2007.60	2007.45	1504.57	1504.07	1504.57
8	3007.57	3007.26	3006.93	1755.61	1755.58	1754.64

[a]Modes used for identification.

sation. A 44-degree-of-freedom planar truss is used in this second demonstration.

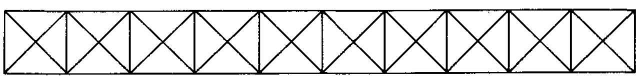
Four optimal-update identification methods, two previously published and two developed in this work, are each applied to the example problem designed by Kabe[3] to demonstrate and compare their performance. Figure 1 shows the system and defines the mass and stiffness properties. Kabe's problem includes stiffness values of various magnitudes and exhibits closely spaced frequencies, both among characteristics expected for large space structures.

Element-by-element stiffness matrix results for each of the four update methods are presented in Table 1. Incorrect values in the original stiffness matrix are presented, along with exact values of the correct result. Each method used the first three modes of the exact model as data to perform the identification. Adjusted stiffness values from MS-DFP (Baruch/Bar Itzhack), MS-BFGS, Kabe's method, and MSMT make up the center columns. Kabe's method and the MSMT method preserve the zero-nonzero pattern of the original stiffness matrix while the other two methods do not. Large nonzero values for elements (3,6) and (5,6) imply direct load paths that do not exist in the physical system. Kabe's method and the MSMT method both correctly identify the stiffness matrix elements.

In a similar format, Table 2 presents the eigenvalue results for the same example to illustrate the influence of the constraint equations. The three modes used for the identification are reproduced as eigensolutions of the updated model from all four methods. However, Kabe's method and the MSMT method also improve the frequency results for all modes due to correct identification of the complete stiffness matrix. The MS-DFP (Baruch/Bar Itzhack) and MS-BFGS show slight improvement in the frequency results for modes 4 and 5, but little change from the incorrect values for modes 6, 7, and 8.

Kabe showed that a minimum of three modes was needed to uniquely identify the stiffness matrix in this example. Preservation of the zero-nonzero pattern of the original model enables complete identification with a small number of observed modes. Performance of the MSMT method is seen to be comparable to that of Kabe's method. Therefore, the MSMT method, with its reduced storage and computational requirements by comparison to Kabe's, seems ideally suited for on-orbit identification applications.

Modal data from actual structures will contain measurement and processing errors. Imposition of this data as a hard constraint in the optimal update methods leads to an updated model that will reproduce the erroneous data as eigensolutions. There may be no update possible with the desired sparsity pattern that imposes consistency with noisy modal data. The error compensation version of the MSMT method was designed to overcome these problems. The second demonstration problem is presented to compare the MSMT-EC performance to MSMT and Kabe's method performance.



L_h = 196.85 in (5 m) Horizontal member length
L_v = 196.85 in (5 m) Vertical member length
E_x = 40 x 10^6 psi Axial elastic modulus
ρ = 0.063 lb/in^3 Density
A = 0.3657 in^2 Member cross-section area

Fig. 2 Planar truss example problem.

A 44-degree-of-freedom planar truss, as shown in Fig. 2, was selected for the second simulated problem. The truss is constructed of members with properties similar to those proposed for the space station design. In this problem, which represents a damage location case, the original model is correct for the undamaged structure. The actual structure has the lower longeron of the fifth bay removed. Modal data for the first three elastic modes of the model of the damaged structure was corrupted with uniformly distributed random errors to generate measured data for this simulation. Each mode shape value was multiplied by $1.0 + \mu$ where μ was a uniformly distributed random variable that varied between ± 0.10. An orthogonality check with respect to the mass matrix revealed that no off-diagonal terms exceeded 0.10, so these corrupted modes represent quality that would be accepted in an actual test. Even so, 10% error is severe and used here to show performance limits, not typical performance quality.

The corrupted modes were orthogonalized with the method of Ref. 1, then identification with Kabe's method, MSMT without error compensation, and MSMT-EC was performed. With exact data, these three modes are sufficient to correctly identify the reduced stiffness values.

Table 3 presents a comparison of the eigenvalue results from the adjusted models for the first seven elastic modes. The original model was improved by all methods. The three measured eigenvalues are matched exactly by the adjusted models. Kabe's method adjusted the unmeasured eigenvalues, improving mode 7, but undershooting modes 8–10. The MSMT and MSMT-EC solutions are identical and slightly better than Kabe's in adjusting the unmeasured eigenvalues presented.

Without orthogonalization of the test modes, the three measured eigenvalues are improved from those of the original model, but not exactly reproduced. Orthogonalization of the test modes also enabled convergence of the MSMT method. Inconsistencies in the corrupted data without orthogonalization prevented convergence of the MSMT method for this case. The error compensating version, MSMT-EC, converged to essentially equivalent solutions whether the data was orthogonalized or not, due to the soft constraint in the formulation. Also, Kabe's method performed better with orthogonalized modes, but not as well as the secant methods.

Rigid body modes for each of the three methods were adjusted from near-zero eigenvalues to nonzero values, including two negative values. Kabe[4] noted that a nonpositive definite stiffness matrix could result in some cases where the data included random errors. The possibility of a nonpositive definite result is consistent with the framework of sparsity-preserving secant methods, as well.

Table 4 is a comparison of the stiffness matrix values of the fifth bay of the truss. Only three elements of the 44×44

Table 3 Planar truss elastic mode eigenvalue results from three methods using corrupted data

Mode number	Original model, $\times 10^5$	Adjusted models, $\times 10^5$ Kabe	MSMT	MSMT-EC	Exact model
4	0.2254	0.1077	0.1077	0.1077	0.1077[a]
5	1.4305	1.3104	1.3104	1.3104	1.3104[a]
6	2.5121	1.8609	1.8609	1.8609	1.8609[a]
7	4.4891	3.7920	3.9038	3.9038	4.0932
8	9.9448	5.9408	6.4742	6.4742	9.2528
9	10.1637	8.1538	8.5822	8.5822	10.1053
10	18.0124	11.0627	14.5674	14.5674	16.4360

[a]Modes used for identification.

Table 4 Stiffness matrix results for the fifth bay of the planar truss from three methods using corrupted data

Element number	Original model, lb/in. $\times 10^5$	Adjusted models,[a] lb/in. $\times 10^5$ Kabe	MSMT	MSMT-EC	Exact model, lb/in. $\times 10^5$	
17,17	2.0116	1.3576	1.4507	1.4507	1.2685	←
17,21	−0.7431	−0.0787	−0.0765	−0.0765	0.0000	←
17,23	−0.2627	−0.3150	−0.3785	−0.3785	−0.2627	
17,24	−0.2627	−0.2636	−0.2822	−0.2822	−0.2627	
18,18	1.2685	1.2034	1.4360	1.4360	1.2685	
18,20	−0.7431	−0.6275	−0.7799	−0.7799	−0.7431	
18,23	−0.2627	−0.1814	−0.1133	−0.1133	−0.2627	
18,24	−0.2627	−0.2532	−0.2691	−0.2691	−0.2627	
19,19	2.0116	1.8854	2.0808	2.0808	2.0116	
19,21	−0.2627	−0.2407	0.0027	0.0027	−0.2627	
19,22	0.2627	0.1894	0.1846	0.1846	0.2627	
19,23	−0.7431	−0.7519	−0.7466	−0.7466	−0.7431	
20,20	1.2685	1.1042	1.1571	1.1571	1.2685	
20,21	0.2627	0.2045	0.2771	0.2771	0.2627	
20,22	−0.2627	−0.2233	−0.1950	0.1950	−0.2627	
21,21	2.0116	0.9726	1.1433	1.1433	1.2685	←
22,22	1.2685	0.9456	1.0391	1.0391	1.2685	
22,24	−0.7431	−0.6055	−0.7352	−0.7352	−0.7431	
23,23	2.0116	1.4777	1.4650	1.4650	2.0116	
24,24	1.2685	1.2822	1.4945	1.4945	1.2685	

[a]Three modes were used by each method for the identification.

symmetric stiffness matrix are affected by the removal of the longeron. Arrows on the far right direct attention to the elements changed by the structure damage. The identified models all show large stiffness reductions for the three highlighted elements with smaller, but nonzero, changes throughout the matrix.

The adjusted models that result from the three methods for the simulated problem with corrupted data are essentially equivalent. Orthogonalization of the data prior to its use in the identification methods improved the eigenvalue results for all methods, with the MSMT-EC method giving better results with or without orthogonalization. Stiffness matrix element results for all methods were comparable and not substantially affected by orthogonalization of the data, except by producing convergence of the MSMT method without error compensation.

More study is needed, however, to determine criteria for data for best identification results. Selection of appropriate weighting values for the MSMT-EC method for its best performance also requires further study. In the framework of secant methods, these issues may be addressed more easily.

The simulated problems demonstrate the MS-DFP (Baruch/Bar Itzhack), MS-BFGS, Kabe's, and MSMT methods. Connectivity is preserved with Kabe's method and the MSMT methods, with and without error compensation. Iterative solution of the auxiliary system of equations in the MSMT methods reduces the storage and computational effort required to preserve connectivity. The MSMT-EC version forms a positive definite auxiliary system, which guarantees convergence for the classical conjugate gradient method.

Summary

Optimal-update structural identification methods for stiffness matrix adjustment with measured data are related to secant methods from quasi-Newton approaches in nonlinear optimization. These secant methods have been investigated extensively for nearly twice as long as optimal-update identification methods, hence, the recognition of a relationship between the two fields immediately advanced understanding of optimal-update identification methods. In addition, new methods for stiffness matrix adjustment were developed by generalizing certain secant methods to include multiple constraints. Of these, the MSMT method overcomes difficulties that have restricted large space structure applications of previously developed methods. Also, an error compensation version of the MSMT method reduces the effect of errors in the measured data.

Two numerical simulations were used to demonstrate the new stiffness matrix identification methods. Results of the MS-BFGS method are comparable to the MS-DFP (Baruch/Bar Itzhack) method. Results with the MSMT method are comparable to results of Kabe's previously published method. The error compensation version of the MSMT produces comparable results as well, without the concerns of the MSMT method without compensation.

Within the framework of secant methods, improved understanding of these structural identification methods and further research on their application to actual structures will enable their use in various identification situations, including on-orbit testing of large space structures.

Appendix

Toint's[14] original analysis is followed here, with generalizations producing a multiple-secant update. The cost functional for the MSMT method,

$$\| [D]^{-1}([K] - [K_c])[D]^{-1} \|_F^2 \tag{A1}$$

includes a diagonal weighting matrix, selected as $[D] = \mathrm{diag}(d_i)$ where $d_i = \sqrt{K_{ii}^c}$. Initially, for presentation convenience, $[D]$ is taken to be the identity matrix $[I]$. Later, a variable change will produce the update for the weighted cost functional.

The objective is to find a matrix $[K_+]$ that solves

$$\min_{[K]} \{ \| [K] - [K_c] \|_F | [K][S] = [M][S][\Omega^2], [K] = [K]^t, \text{ sparse }([K]) = \text{sparse }([K_c]) \} \tag{A2}$$

Defining $[E] = [K] - [K_c]$ and $[Y] = [M][S][\Omega^2] - [K_c][S]$, Eq. (A2) is rewritten as

$$\min_{[E]} \{ \| [E] \|_F | [E][S] = [Y], [E] = [E]^t, \text{ sparse } [E] = \text{sparse } [K_c] \} \tag{A3}$$

The first and last constraints are combined as

$$\sum_{j=1}^{n} E_{ij} S_{jk}^{(i)} = Y_{ik} \qquad \text{for } i = 1, \dots, n \text{ and } k = 1, \dots, p \tag{A4}$$

where

$$S_{jk}^{(i)} = \begin{cases} S_{jk} & \text{if } j \in \mathrm{adj}(i) \\ 0 & \text{otherwise} \end{cases}$$

The column vector $\{S_{jk}^{(i)}\}_{j=1,\dots,n}$ is the kth column of $[S]$ masked by the zero-nonzero pattern of the ith row of $[K_c]$. The remaining constraint $[E] = [E]^t$ is made implicit by defining

$$[E] = ½([B] + [B]^t) \tag{A5}$$

for an arbitrary matrix $[B]$.

The final formulation is then

$$\min_{[B]} ½ \| ½([B] + [B]^t) \|_F^2 \tag{A6}$$

subject to

$$\sum_{j=1}^{n} (B_{ij} + B_{ji}) S_{jk}^{(i)} = 2Y_{ij} \qquad \text{for } i = 1, \dots, n \text{ and } k = 1, \dots, p$$

Applying the method of Lagrange multipliers, the Lagrangian function is

$$L(B, \lambda) = \frac{1}{8} \sum_{i,j=1}^{n} (B_{ij} + B_{ji})^2 + \sum_{i=1}^{n} \sum_{k=1}^{p} \lambda_{ik} \left[2Y_{ik} - \sum_{j=1}^{n} (B_{ij} + B_{ji}) S_{jk}^{(i)} \right] \tag{A7}$$

where λ_{ik} are the Lagrange multipliers. Taking the partial derivative with respect to each element B_{rs} yields, after simplification, the final update equation,

$$E_{rs} = ½(B_{rs} + B_{sr}) = \sum_{k=1}^{p} (\lambda_{rk} S_{sk}^{(r)} + \lambda_{sk} S_{rk}^{(s)}) \tag{A8}$$

Recall that $K_{rs}^+ = K_{rs}^c + E_{rs}$. The partial derivative of the Lagrangian function with respect to each Lagrange multiplier is set equal to zero to establish a system of linear equations that is used to solve for the Lagrange multipliers needed for the update,

$$\sum_{\ell=1}^{p} \left(\sum_{j=1}^{n} S_{j\ell}^{(i)} S_{jk}^{(i)} \right) \lambda_{i\ell} + \sum_{j=1}^{n} S_{jk}^{(i)} \sum_{\ell=1}^{n} S_{i\ell}^{(j)} \lambda_{j\ell} = Y_{ik} \tag{A9}$$

To express this equation in matrix form, two partitioned column vectors are defined $\{\Gamma\}$ and $\{\Delta\}$, where the subvectors are

$$\{\Gamma_i\} = \begin{bmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{ip} \end{bmatrix} \qquad \{\Delta_i\} = \begin{bmatrix} Y_{i1} \\ Y_{i2} \\ \vdots \\ Y_{ip} \end{bmatrix} \tag{A10}$$

In addition, $[P_i]$ is created as a diagonal projection matrix to mask the mode shape vectors with the zero-nonzero pattern of the ith row of $[K_c]$ so

$$([P]_i [S])_{jk} = S_{jk}^{(i)} \tag{A11}$$

The resulting linear system that is solved for the Lagrange multipliers becomes Eq. (11) (repeated here for convenience)

$$[F]^t([I] + [\Pi])[F] \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{bmatrix} \tag{A12}$$

For the nonweighted cost functional,

$$[F] = \mathrm{diag} \begin{bmatrix} [P]_1[S] & & & \\ & [P]_2[S] & & \\ & & \ddots & \\ & & & [P]_n[S] \end{bmatrix} \tag{A13}$$

With the weighting matrix $[D]$ as originally defined, $[F]$ and $\{\Delta_i\}$ are as presented in the main text, Eqs. (12) and (13), respectively.

The conjugate gradient approach requires the matrix-vector product

$$[F]^t([I] + [\Pi])[F] \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix} \tag{A14}$$

for each iterate. However, the product

$$[F] \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix} = \begin{bmatrix} P_1 S \; \Gamma_1 \\ P_2 S \; \Gamma_2 \\ \vdots \\ P_n S \; \Gamma_n \end{bmatrix} \tag{A15}$$

for the nonweighted formulation can be stored in an $n \times n$ matrix $[T]$ as

$$[T] = [P_1 S \Gamma_1 \; P_2 S \Gamma_2 \; \dots \; P_n S \Gamma_n] \tag{A16}$$

where $[T]$ has the same sparsity structure as $[K_c]$ though it is not symmetric. Then

$$([I] + [\pi])[F] \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{bmatrix} \tag{A17}$$

is $[T] + [T]^t$, which is symmetric and has the same sparsity structure as $[K_c]$. With ϕ_i as the ith column of $[T] + [T]^t$, the next iterate is finally formed as

$$\Gamma_i^+ = ([P]_i[S])^t\phi_i \tag{A18}$$

Therefore, the solution is achieved through iteration and storage equivalent to that for the original stiffness matrix.

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